

STATS 2D03 Assignment 1

Name:

Student Number:

1. The lifetime of a machine part has a continuous distribution on the interval $(0, 40)$ with probability density function $f(x)$, where $f(x)$ is proportional to $(10+x)^{-2}$ on the interval. Calculate the probability that the lifetime of the machine part is less than 6. (2%)
2. A study is being conducted in which the health of two independent groups of ten policyholders is being monitored over a one-year period of time. Individual participants in the study drop out before the end of the study with probability 0.2 (independently of the other participants). Calculate the probability that at least nine participants complete the study in one of the two groups, but not in both groups? (2%)
3. A company takes out an insurance policy to cover accidents that occur at its manufacturing plant. The probability that one or more accidents will occur during any given month is 0.60. The numbers of accidents that occur in different months are mutually independent. Calculate the probability that there will be at least four months in which no accidents occur before the fourth month in which at least one accident occurs. (2%)
4. Let X be a continuous random variable with density function

$$f(x) = \begin{cases} \frac{|x|}{10}, & -2 \leq x \leq 4 \\ 0, & \text{otherwise} \end{cases}. \text{ Calculate the expected value of } X. \text{ (2\%)}$$

5. A random variable X has the cumulative distribution function

$$F(x) = \begin{cases} 0, & x < 1 \\ \frac{x^2 - 2x + 2}{2}, & 1 \leq x < 2 \\ 1, & x \geq 2 \end{cases}. \text{ Calculate the variance of } X. \text{ (2\%)}$$